

## Duane-Hunt displacement law and Planck's "quantum of action" 5.4.09-00



## What you can learn about ...

- X-ray tube
- Bremsstrahlung
- Characteristic X-ray radiation
- Energy levels
- Crystal structures
- Lattice constant
- Interference
- The Bragg equation

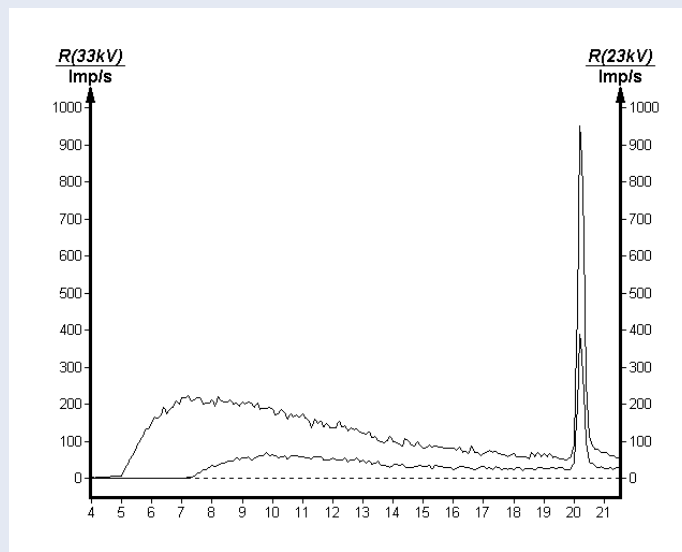
## Principle:

X-ray spectra are to be recorded as a function of the anode voltage. The short wavelength limit of the bremspectrum is to be used to determine the agreement with the Duane-Hunt displacement law, and to determine Planck's "quantum of action".

## What you need:

X-ray basic unit, 35 kV	09058.99	1
Goniometer for X-ray unit, 35 kV	09058.10	1
Plug-in module with Cu X-ray tube	09058.50	1
Counter tube, type B, BNC cable, $l = 50$ cm	09005.00	1
Lithium fluoride crystal, mounted	09056.05	1
Software X-ray unit, 35 kV	14407.61	1
Data cable 2 x SUB-D, plug/socket, 9 pole	14602.00	1
PC, Windows® XP or higher		

Complete Equipment Set, Manual on CD-ROM included  
 Duane-Hunt displacement law and  
 Planck's "quantum of action" P2540900



Bremsstrahlung as function of two anode voltages; Glancing angle  $\vartheta$  as x-axis in degree.

## Tasks:

1. The intensity of the X-rays emitted by the copper anode at various anode voltages is to be recorded as a function of the Bragg angle, using a LiF monocrystal.
2. The short wavelength limit (= maximum energy) of the bremspectrum is to be determined for the spectra obtained in (1).
3. The results are to be used to verify the Duane-Hunt displacement law, and to determine Planck's "quantum of action".

**Related topics**

X-ray tube, bremsstrahlung, characteristic X-rays, energy levels, crystal structures, lattice constant, interference, the Bragg equation.

**Principle**

X-ray spectra are to be recorded as a function of the anode voltage. The short wavelength limit of the bremspectrum is to be used to determine the agreement with the Duane-Hunt displacement law, and to determine Planck's "quantum of action".

**Equipment**

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Goniometer for X-ray unit, 35 kV	09058.10	1
Plug-in module with Cu X-ray tube	09058.50	1
Counter tube, type B	09005.00	1
Lithium fluoride crystal, mounted	09056.05	1
Recording equipment:		
XYt recorder	11416.97	1
Connecting cable, $l = 100$ cm, red	07363.01	2
Connecting cable, $l = 100$ cm, blue	07363.04	2
or		
Software X-ray unit, 35 kV	14407.61	1
Data cable, 2 x SUB-D, plug/socket, 9 pole	14602.00	1
PC, Windows® 95 or higher		

**Tasks**

1. The intensity of the X-rays emitted by the copper anode at various anode voltages is to be recorded as a function of the Bragg angle, using an LiF monocrystal.

2. The short wavelength limit (= maximum energy) of the bremspectrum is to be determined for the spectra obtained in (1).
3. The results are to be used to verify the Duane-Hunt displacement law, and to determine Planck's "quantum of action".

**Set-up and procedure**

Set up the experiment as shown in Fig. 1. Fix the diaphragm tube with 1 mm diameter aperture in the X-ray outlet tube. With the X-ray unit switched off, connect the goniometer and the counter tube to the appropriate sockets in the base plate of the experimenting area. Set the goniometer block with mounted analyzing crystal to the middle position, and the counter tube to the right stop position.

The following settings are recommended for the recording of the spectra:

- Auto and Coupling mode
- Gate time 2 s: Angle step width  $0.1^\circ$
- Anode current  $I_A = 1$  mA
- Anode voltages  $33$  kV  $> U_A > 13$  kV;  $\Delta U_A = 2$  kV

When the spectra are to be recorded with an XY recorder, connect the Y axis to the analog output (Imp/s) of the X-ray basic unit and, correspondingly, the X input to the analog output for the angular position of the crystal (select the analog signal for the crystal angle with the selection button for this output). When a PC is to be used for recording purposes, connect it via the SUB-D socket of the X-ray basic unit.

Fig. 1: Experimental set-up for the analysis of X-ray energies



Use the LiF monocrystal as analyzer to first record a general spectrum at maximum values of anode voltage and current (Fig. 2). Subsequent to this, record spectra of the bremsstrahlung up to the  $K_{\beta}$ -line at various anode voltages (Fig. 3). When an XY recorder is used, increase the sensitivity of the X channel accordingly and calibrate carefully, otherwise an exact determination of the angle of the starting point of the bremsstrahlung will not be possible.

**Note**

Never expose the counter tube to primary radiation for a longer length of time.

**Theory and evaluation**

The positive voltage  $U_A$  lying on the cathode of the X-ray tube causes the electrons which are emitted from the cathode with only a rather low energy distribution to accelerate. On reaching the cathode, the electrons have the energy:

$$E_{kin} = eU_A \text{ (where } e = \text{the elementary charge)} \quad (1)$$

A portion of the electrons will be progressively slowed down on arrival, and their kinetic energy so be converted to electromagnetic radiation with a continuous energy distribution. Some electrons, however, will have their kinetic energy converted into electromagnetic radiation in a single step, so giving the bremsstrahlung a short wavelength limit. In 1915, Duane and Hunt found empirically that the product of the anode voltage and the shortest wavelength  $\lambda_{min}$  is a constant, and that the following relationship holds:

$$U_A \cdot \lambda_{min} \propto 1.25 \cdot 10^{-6} \text{ V} \cdot \text{m} \quad (2)$$

This relationship can easily be derived from Einstein's energy equation, according to which:

$$E_{kin} = e \cdot U_A = h \cdot f_{max} = h \frac{c}{\lambda_{min}} \quad (3)$$

Where Planck's constant  $h = 6.6256 \cdot 10^{-34} \text{ Js}$   
 The velocity of light  $c = 2.9979 \cdot 10^8 \text{ m/s}$   
 The elementary charge  $e = 1.6021 \cdot 10^{-19} \text{ As}$

The calculation for the shortest wavelength becomes:

$$\lambda_{min} = 12.398 \cdot 10^{-5} \frac{1}{U_A} \text{ V} \cdot \text{m}$$

The analysis of the polychromatic X-rays is carried out using a monocrystal. If the X-rays impinge under a glancing angle  $\vartheta$ , constructive interference will only appear in reflections if the paths of the partial waves reflected on the lattice planes differ by one or more wavelengths. This situation is described by the Bragg equation:

$$2d \sin \vartheta = n\lambda \quad (4)$$

(interplanar spacing  $d = 201.4 \text{ pm}$ ;  $n = 1, 2, 3, \dots$ )

Fig. 3: Bremsstrahlung as function of two anode voltages; Glancing angle  $\vartheta$  as x-axis in degree

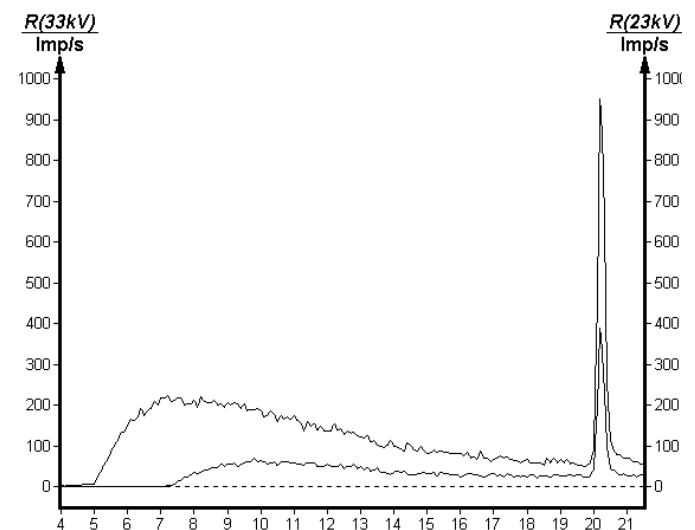
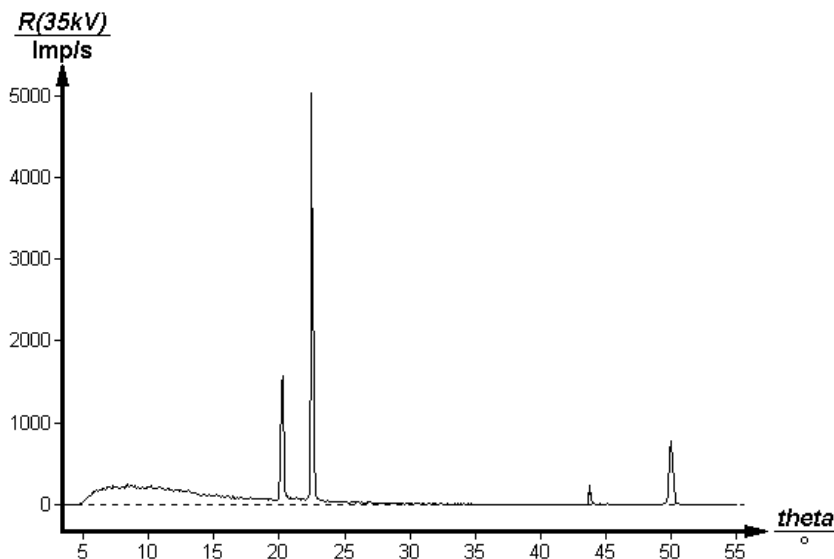


Fig. 2: Copper X-ray spectrum, using a LiF analyzer



The short wavelength limit of the bremsstrahlung is determined by the appertaining glancing angle  $\vartheta$ . The value of  $\lambda_{\min}$  can be calculated using (4).

In Fig. 4.,  $\lambda_{\min}$  is represented as a function of the reciprocal value of the anode voltage  $1/U_A$ . The slope of the resulting straight line is:

$$m = \frac{\lambda_{\min}}{1/U_A} = (12.42 \pm 0.07) \cdot 10^{-5} \text{ V} \cdot \text{m}$$

This result is acceptably consistent with the Duane-Hunt displacement law.

Planck's "quantum of action" (Planck's constant) can be determined using the same measured curves.

From (3) and (4), we obtain:

$$U_A = \frac{h \cdot c}{2e \cdot d \cdot \sin\vartheta} \quad (5)$$

In Fig. 5  $\sin \vartheta$  is plotted as a function of the reciprocal value of the anode voltage  $1/U_A$ . Using the slope of the resulting line, we obtain:

$$h = m \frac{2e \cdot d}{c} = (6.64 \pm 0.04) \cdot 10^{-34} \text{ Js}; \frac{\Delta h}{h} = \pm 0.6\%$$

In conjunction with (5), and with the values for the glancing angle found from Fig. 2, the energies of the characteristic copper X-ray lines can be calculated to be:

$n = 1$	$E_{-K\alpha} = 8.009 \text{ keV}$
	$E_{-K\beta} = 8.83 \text{ keV}$
$n = 2$	$E_{-K\alpha} = 8.012 \text{ keV}$
	$E_{-K\beta} = 8.873 \text{ keV}$

These values are in good agreement with the literature values (see also experiment P2540100).

### Recommendation

For the evaluation of the experiments performed with the software program "X-ray spectroscopy" you first have to convert the glancing angles  $\vartheta$  (crystal angle = x-axis) of the spectra into the equivalent wavelength values.

To do this, you have to perform the following steps:

1. „Analysis“ – “X-ray spectroscopy“ – “Convert x-axis – “Wave length (Calculate)“.

Based on this modified spectrum (Imp/s  $\leftrightarrow$  f(lambda/pm) you can obtain the Hunt-straight line performing the following steps:

2. Mark the following option consecutively: “Analysis“ – “X-ray spectroscopy“ – “Duane-Hunt- straight line“.

The window “Duane-Hunt- straight line“ will pop up. You have to mark the onset point of the Bremsspectrum and click on “Accept value“. An additional window will show you the anode voltage and the wavelength.

This method has to be repeated for all spectra with different anode voltages.

To visualise the graph click on the button “Generate Duane-Hunt- straight line“.

Finally Planck's quantum of action is calculated by clicking the buttons

“Analysis“ – “X-ray spectroscopy“ – “Determine Planck-constant“.

With the “Display options“ – “Channels“ and “Symbol“ you may add the measuring points within the graph of the “Duane-Hunt- straight line“.

Fig. 4: Duane-Hunt displacement law;  $\lambda_{\min}$  as a function of the reciprocal of the anode voltage  $1/U_A$

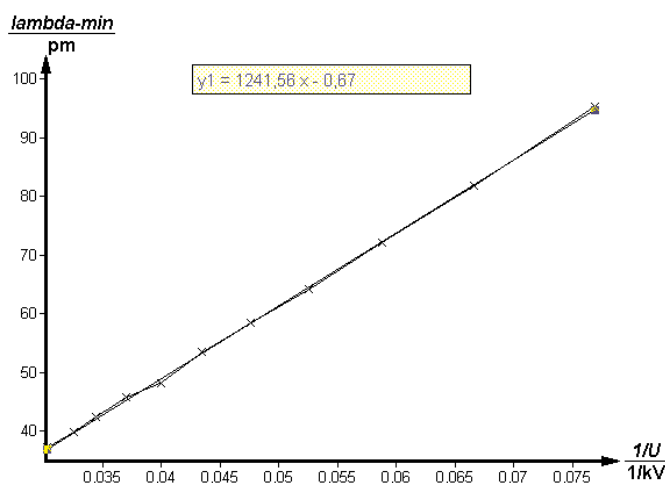


Fig. 5: Planck's "quantum of action";  $\sin\vartheta$  as a function of the reciprocal of the anode voltage  $1/U_A$ .

